Solar cycle 25 prediction using length-to-amplitude relations

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Abstract. We propose a simple method for prediction of the 11-year solar cycle maximum that is based on two relations. One of them is well known Waldmeier's rule that binds the amplitude of a cycle and the length of its ascending phase. The second rule relates the length of a given cycle from minimum to minimum and the amplitude of the next one. Using corresponding linear regressions we obtain for the amplitide of cycle 25 in the scale of 13-month smoothed monthly total revised sunspot number $\mathrm{SN}_{\max}(25) = 181 \pm 46$ and for the moment of the maximum $T_{\max}(25) = 2024.2 \pm 1.0$. Therefore, according to the prediction, cycle 25 will be higher than the previous one ($\mathrm{SN}_{\max}(24) = 116$) with probability 0.92.

Keywords: solar cycle prediction; cycle 25; Waldmeier rule

(Du et al., 2022) Du et al., 2022

1. Introduction

In 2019 experts of Solar Cycle 25 Prediction Panel predicted that the maximum of the solar cycle 25 (SC25) will lie in the sunspot number range between 95 and 130, that is similar to SC24 with the maximal SN=116 (`https://www.weather.gov/news/190504-sun-activity-in-solar-cycle`). The opinion that SC25 will be equal or lower than SC24 is shared by many authors (Du et al., 2022; Burud et al., 2021; Nandy, 2021; Wu and Qin, 2021; Courtillot et al., 2021 etc).

If the odd SC25 is lower than the even SC24, it will violate the Gnevyshev-Ohl correlation rule (Gnevyshev and Ohl, 1948). Many regard this as a sign that the Sun is entering to a Dalton-like global minimum.

However, there is an alternative point of view that the forecast of the experts has been underestimated and the forecoming cycle will be higher (McIntosh et al., 2020; Koutchmy et al., 2021; Prasad et al., 2022; Lu et al., 2022 etc). None of the two positions have prevailed so far.

In the following we propose a simple method of solar maximums predictions based on linear relations between amplitudes of cycles and lengths of its phases. This method provides one more argument that SC25 will be higher that SC24 and comparable with SC23.

2. Data and notation

Hereafter we will use for analysys and prediction the 13-month smoothed monthly averages of the recalibrated sunspot number SN for 1749–2021 (Clette et al., 2014) (`https://wwwbis.sidc.be/silso/DATA/SN_ms_tot_V2.0.txt`).

We will introduce the followig notation: $\mathrm{SN}_{\max}(i)$ is the sunspot index in the maximum of the ith cycle (i.e. its amplitude), $T_{\min}(i)$ and $T_{\mathrm{Max}}(i)$ are moments of its minimum and maximum, $T_{\mathrm{mm}}(i) = T_{\min}(i+1) - T_{\min}(i)$ is its length (from minimum to minimum), $T_{\mathrm{mM}}(i) = T_{\mathrm{Max}}(i) - T_{\min}(i)$ is the length of the ascending branch of the cycle (from minimum to maximum).

3. The Waldmeiers rule

The well-known empirical Waldmeier rule (WR) states that the length of the ascending branch of the cycle anticorrelates with its amplitude (Waldmeier, 1935) (Fig. 1). One can find the linear regression equation

$$\begin{aligned} \mathrm{SN}_{\max}(i) &= a_1 + b_1\, T_{\mathrm{mM}}(i)\,, \\ a_1 &= 340.2 \pm 31.3\,, \\ b_1 &= (-36.76 \pm 6.90)\ \mathrm{yr}^{-1}\,, \end{aligned} \tag{1}$$

and the correlation coefficient between the regression parameters is

$$r(a_1, b_1) = -0.976\,.$$

4. The length-to-next-amplitude rule (LNAR)

Another empirical rule, similar to WR but less known (and still nameless, to the best of our knowledge), states that the length of a given cycle anticorrelates with the amplitude of the next one (Hathaway et al., 1994; Solanki et al., 2002; Hathaway, 2015; Ivanov, 2021) (Fig. 2). The corresponding regression is

$$\begin{aligned} \mathrm{SN}_{\max}(i+1) &= a_2 + b_2\, T_{\mathrm{mm}}(i)\,, \\ a_2 &= 544.5 \pm 86.6\,, \\ b_2 &= (-33.01 \pm 7.81)\ \mathrm{yr}^{-1}\,, \\ r(a_2, b_2) &= -0.994\,. \end{aligned} \tag{2}$$

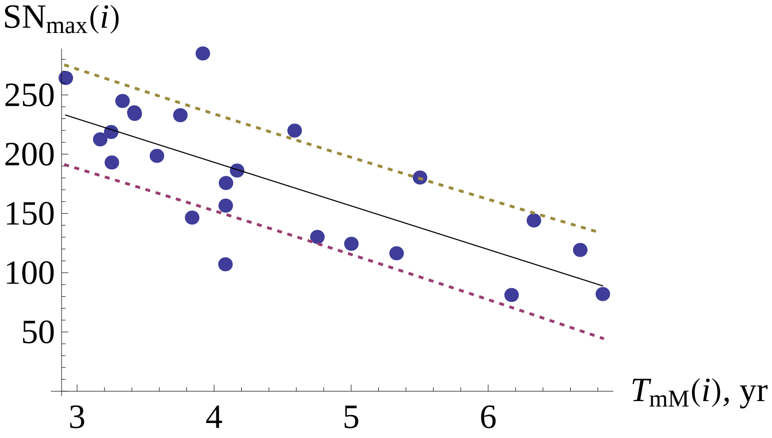

Figure 1. The relationship between the length of the ascending phase $T_{mM}(i)$ and the amplitude of the cycle $SN_{max}(i)$ (the Waldmeier rule). The correlation coefficient is $r = -0.75$. The solid curve is the linear regression (1). The dashed curves mark the standard error ranges for predictions.

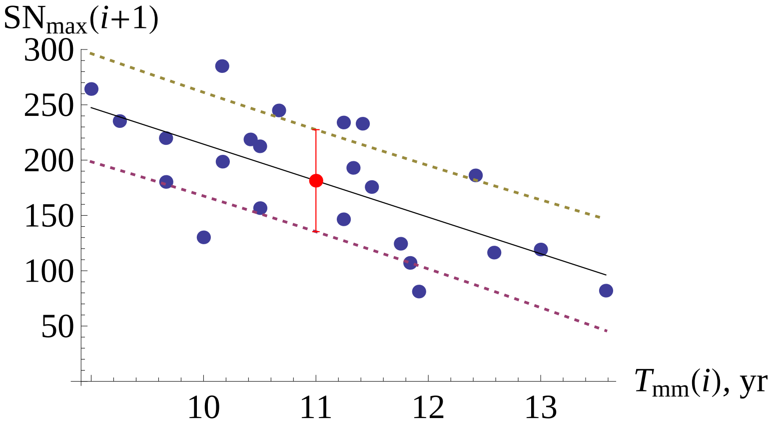

Figure 2. The relationship between the length of the cycle $T_{mm}(i)$ and the amplitude of the next one $SN_{max}(i+1)$ (the LNA rule). The correlation coefficient is $r = -0.68$. The solid curve is the linear regression (2). The red circle is the prediction for SC25. The dashed curves mark the standard error ranges for predictions.

5. The length-to-ascending-length rule (LALR)

A direct consequence of the WR and LNAR is the third rule, that binds the length of the current cycle with the length of the ascending phase of the next one (Fig. 3). These parameters correlate, and the regression is

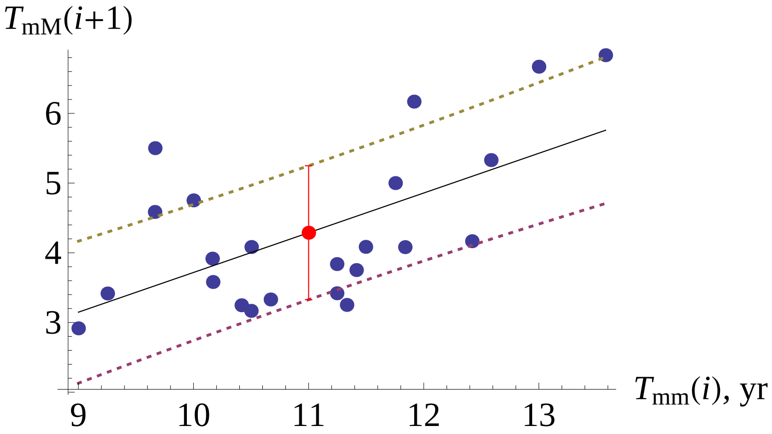

Figure 3. The relationship between the length of the cycle $T_{\rm mm}(i)$ and the length of the next ascending phase $T_{\rm mM}(i+1)$ (the LAL rule). The correlation coefficient is $r = +0.61$. The red circle is the prediction for SC25. The solid curve is the linear regression (3). The dashed curves mark the standard error ranges for predictions.

$$\begin{aligned} \mathrm{T}_{\rm mM}(i+1) &= a_3 + b_3\, T_{\rm mm}(i)\,, \\ a_3 &= (-1.98 \pm 1.81)\ \mathrm{yr}\,, \\ b_3 &= 0.570 \pm 0.163\,, \\ r(a_3, b_3) &= -0.994\,. \end{aligned} \tag{3}$$

6. Prediction of SC25

The LNA and LAL rules stated above allow to estimate the time and magnitude of the maximum of the next cycle provided we know the length of the current one. Since the minimum between SC24 and SC25 in the 13-month smoothed index occurs in December 2019, the length of the SC24 is 11.0 yr, and it follows from (1) and (3) that

$$\mathrm{SN}_{\rm max}(25) = 181 \pm 46\,,$$

$$\mathrm{T}_{\rm mM}(25) = (4.23 \pm 0.96)\ \mathrm{yr}\,,$$

and

$$T_{\rm max}(25) = (2024.24 \pm 0.96)\ \mathrm{yr}$$

(see Figs. 2 and 3).

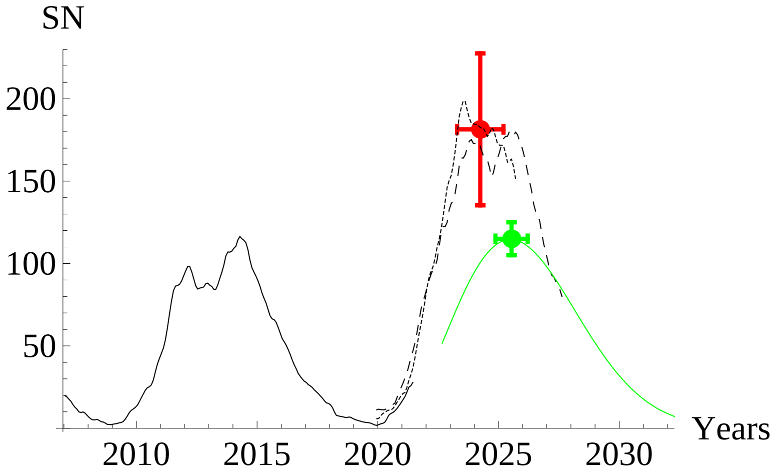

Figure 4. The observed SN index for SC24 and SC25 (black line), SC17 (short dashes) and SC23 (long dashes) shifted to match the minimums to that of SC25, our prediction of SC25 maximum (red point with errorbars), the prediction of SC25 (green line) and its maximum (green point with errorbars) by the Solar Cycle Prediction Panel (`https://services.swpc.noaa.gov/json/solar-cycle/predicted-solar-cycle.json`).

The errors of the predictions are large enough. Nevertheless, according to the prediction, with probability 0.92 SC25 will be higher than SC24 ($\mathrm{SN}_{\mathrm{max}}(24) = 116$).

Probably, it will be similar to SC23 ($\mathrm{SN}_{\mathrm{max}}(23) = 176$) or SC17 ($\mathrm{SN}_{\mathrm{max}}(17) = 189$) (Fig. 4). It means that the odd SC25 will be higher than the even SC24, and the Gnevyshev-Ohl rule will keep valid.

Earlier (Ivanov, 2021), using the same method, we have obtained for SC25 weaker estimates: the amplitude 136 ± 36 and the moment of maximum 2025.7 ± 0.7 (for the Gaussian smoothing of SN with $\sigma = 8$ months). This underestimate can be explained by the fact that in January 2021, when the mentioned article was being prepared, it was difficult to determine the moment of SC25 minimum in the smoothed index accurately, and we assumed it to occur too late, in October 2020, when the last local minimum of the index before its fast growth took place.

7. Control of stability

To control the stability of the prediction method we will do the following procedure. Let's take the subseries of parameters from 1st to $(i-1)$th cycle, construct the regressions for the LNAR and LALR using the truncated series and obtain the predictions for the moment and amplitude of the ith cycle.

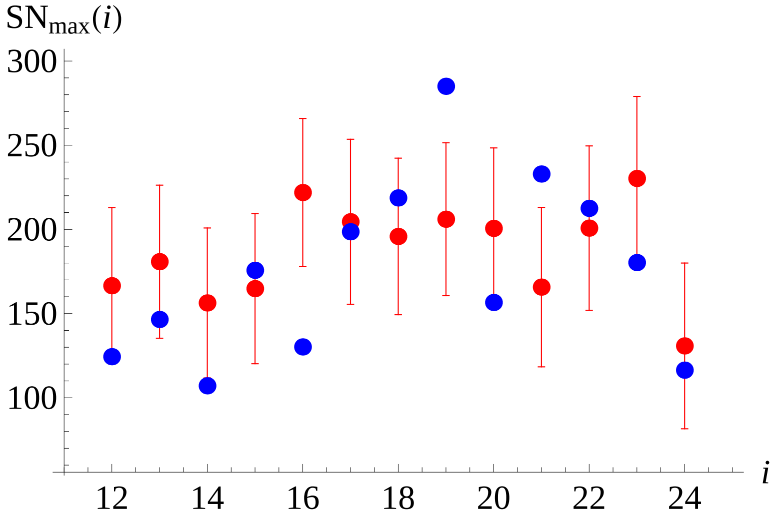

Figure 5. The control predictions for SN_{max} on truncated series. The red circles with errorbars are predictions of the ith SC. The blue circles are observed values.

The results of the predictions compared with the observed values are plotted in Figs. 5 and 6. All predicted values are in 1.5σ bands relative to the observed values. The number of predictions in the one sigma band is 8 out of 13 (62%) for amplitudes and 11 out of 13 (85%) for lengths, which does not contradict to the probability to be in the 1σ band for the normal distribution (68%).

8. Conclusions

The proposed method of prediction, despite its simplicity and rather large errors, provides an accuracy of about 15–30% for the moment of maximum and 25–50% in its magnitude, which is enough to distinguish between cycles of low, medium and high power. In particular, the method predicts SC25 of medium magnitude $SN_{max}(25) = 181 \pm 46$, which will be higher than the previous one ($SN_{max}(24) = 116$) with probability 0.92, and with the same probability the Gnevyshev-Ohl correlation rule will be valid for SC25.

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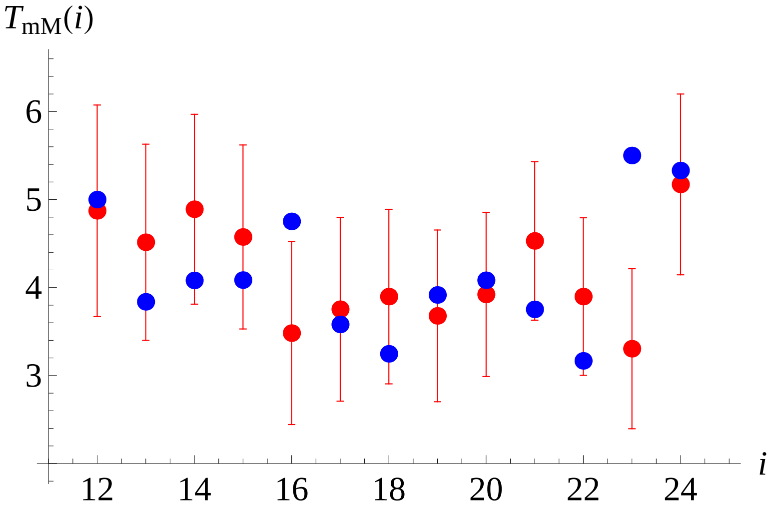

Figure 6. The control predictions for $T_{\rm mM}$ on truncated series. The red circles with errorbars are predictions of the ith SC. The blue circles are observed values.

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